Experimental Violation of Bell Inequality beyond Cirel'son's Bound

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The correlations between two qubits belonging to a three-qubit system can violate the Clauser-Horne-Shimony-Holt-Bell inequality beyond Cirel'son's bound [A. Cabello, Phys. Rev. Lett. **88**, 060403 (2002)]. We experimentally demonstrate such a violation by 7 standard deviations by using a three-photon polarization-entangled Greenberger-Horne-Zeilinger state produced by Type-II spontaneous parametric down-conversion. In addition, using part of our results, we obtain a violation of the Mermin inequality by 39 standard deviations.

As stressed by Peres [1], Bell inequalities [2, 3] have nothing to do with quantum mechanics. They are constraints imposed by local realistic theories on the values of linear combinations of the averages (or probabilities) of the results of experiments on two or more separated systems. Therefore, when examining data obtained in experiments to test Bell inequalities, it is legitimate to do it from the perspective (i.e., under the assumptions) of local realistic theories, without any reference to quantum mechanics. This approach leads to some apparently paradoxical results. A remarkable one is that, while it is a standard result in quantum mechanics that no quantum state can violate the Clauser-Horne-Shimony-Holt (CHSH) Bell inequality [4] beyond Cirel'son's bound, namely $2\sqrt{2}$ [5], the correlations between two qubits belonging to a three-qubit system can violate the CHSH-Bell inequality beyond $2\sqrt{2}$ [6]. In particular, if we use a three-qubit Greenberger-Horne-Zeilinger (GHZ) state [7], we can even obtain the maximum allowed violation of the CHSH-Bell inequality, namely 4 [6].

In this Letter, we report the first observation of a violation of the CHSH-Bell inequality beyond Cirel'son's bound by using a three-photon polarization-entangled GHZ state produced by Type-II spontaneous parametric down-conversion. In addition, since the experiment also provides all the data required for testing Mermin's three-party Bell inequality [8], we use our results to demonstrate the violation of this inequality.

The main idea behind the CHSH-Bell inequality [4] is that, in local realistic theories, the absolute value of a particular combination of correlations between two distant particles i and j is bounded by 2:

$$|C(A, B) - mC(A, b) - nC(a, B)$$
$$- mnC(a, b)| \leqslant 2 \qquad (1)$$

where m and n can be either -1 or 1, and A and a (B and b) are physical observables taking values -1 or 1, referring to local experiments on particle i (j). The

correlation $C(A, B)$ of A and B is defined as

$$C(A, B) = P_{AB}(1, 1) - P_{AB}(1, -1)$$
$$- P_{AB}(-1, 1) + P_{AB}(-1, -1), \qquad (2)$$

where $P_{AB}(1, -1)$ denotes the joint probability of obtaining $A = 1$ and $B = -1$ when A and B are measured.

Cirel'son proved that, for a two particle system prepared in any quantum state, the absolute value of the combination of quantum correlations appearing in the inequality (1) is bounded by $2\sqrt{2}$ [5]. However, assuming local realistic theories' point of view, the correlations predicted by quantum mechanics between two distant qubits belonging to a three-qubit system can violate the CHSH-Bell inequality beyond Cirel'son's bound [6].

In our experiment, the three distant qubits are polarization-entangled photons prepared in the GHZ state:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|H\rangle|H\rangle|H\rangle + |V\rangle|V\rangle|V\rangle), \qquad (3)$$

where H (V) denotes horizontal (vertical) linear polarization. During the experiment, we will analyze the polarization of each photon in one of two different basis: either in the X basis, which is defined as the linear polarization basis H/V rotated by 45^0, which is denoted as H'/V'; or in the Y basis, which is defined as the circular polarization basis R/L (right-hand/left-hand). These polarization bases can be expressed in terms of the H/V basis as

$$|H'\rangle = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle), \quad |V'\rangle = \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle),$$
$$|R\rangle = \frac{1}{\sqrt{2}}(|H\rangle + i|V\rangle), \quad |L\rangle = \frac{1}{\sqrt{2}}(|H\rangle - i|V\rangle). \quad (4)$$

The measurement results H' (R) and V' (L) are denoted by 1 and -1, respectively.

To generate the three-photon GHZ state (3) we use the technique developed in previous experiments [9, 10].
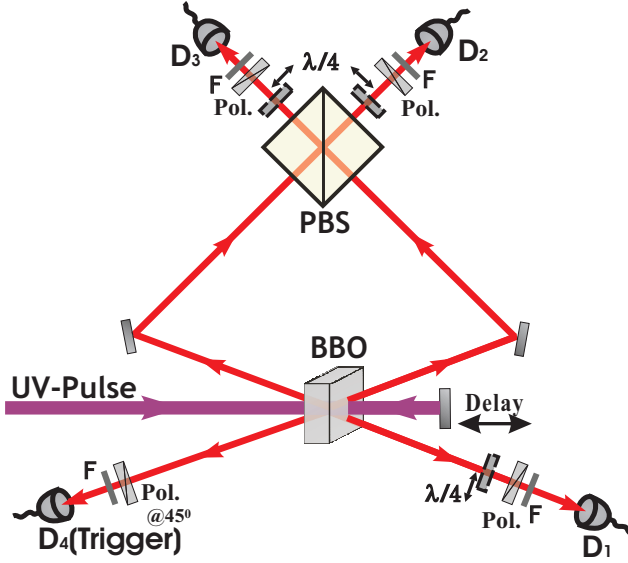
FIG. 1: Experimental setup for generating three-photon GHZ states. An UV pulse passes twice through the BBO crystal to generate two pairs of polarization-entangled photons by Type-II spontaneous parametric down-conversion used to perform the preparation of three-photon GHZ state. The UV laser with a central wavelength of 394 nm has pulse duration of 200fs, a repetition rate of 76 MHz, and an average pump power of 400 mW. We observe about 2×10^4 entangled pairs per second behind 3.6 nm filters (F) of central wavelength 788 nm. Polarizers (Pol.) and quarter wave plates ($\lambda/4$) in front of the detectors are used for performing X or Y measurement.

The experimental setup for generating three-photon entanglement is shown in Fig. 1. A pulse of ultraviolet (UV) light passes through a beta-barium borate (BBO) crystal twice to produce two polarization-entangled photon pairs, where both pairs are in the state

$$|\Psi_2\rangle = 1/\sqrt{2}\left(|H\rangle|H\rangle + |V\rangle|V\rangle\right). \quad (5)$$

One photon out of each pair is then steered to a polarization beam splitter (PBS) where the path lengths of each photon have been adjusted (by scanning the delay position) so that they arrive simultaneously. After the two photons pass through the PBS, and exit it by a different output port each, there is no way whatsoever to distinguish from which emission each of the photons originated, then correlations due to four-photon GHZ entanglement

$$|\Psi_4\rangle = 1/\sqrt{2}\left(|H\rangle|H\rangle|H\rangle|H\rangle + |V\rangle|V\rangle|V\rangle|V\rangle\right) \quad (6)$$

can be observed [10]. After that, by performing a $|H'\rangle$ polarization projective measurement onto one of the four outputs, the remaining three photons will be prepared in the desired GHZ state (3).

In the experiment, the observed fourfold coincident rate of the desired component $HHHH$ or $VVVV$ is
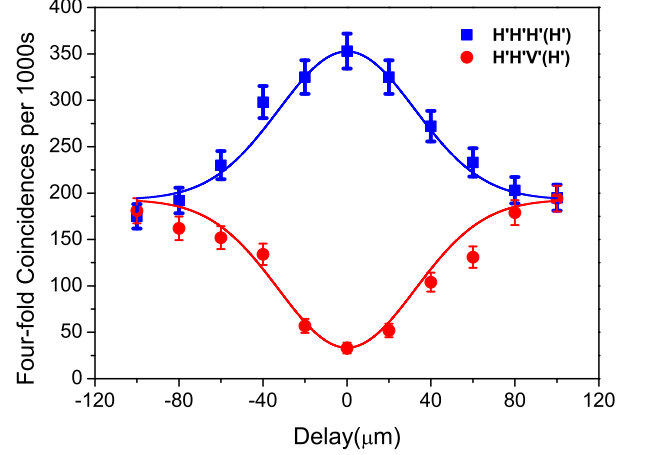
FIG. 2: Typical experimental results for polarization measurements on all three photons in a X basis triggered by the photon 4 at the H' polarization. The coincidence rates of $H'H'H'$ and $H'H'V'$ components are shown as a function of the pump delay mirror position. The high visibility obtained at zero delay implies that three photons are indeed in a coherent superposition.

about 1.4 per second. By performing a H' projective measurement at photon 4 as the trigger of the fourfold coincident, the ratio between any of the desired events HHH and VVV to any of the 6 other nondesired ones, e.g., HHV, is about 65 : 1. To confirm that these two events are indeed in a coherent superposition, we have performed polarization measurements in X basis. In Fig. 2, we compare the count rates of $H'H'H'$ and $H'H'V'$ components as we move the delay mirror (Delay) by the trigger photon 4 at the H' polarization. The latter component is suppressed with a visibility of 83% at zero delay, which confirms the coherent superposition of HHH and VVV.

For each three-photon system prepared in the state (3), we will define as photons i and j those two giving the result -1 when making X measurement on all three photons; the third photon will be denoted as k. If all three photons give the result 1, photons i and j could be any pair of them. Since no other combination of results is allowed for the state (3), i and j are well defined for every three-photon system.

We are interested in the correlations between two observables, A and a, of photon i and two observables, B and b, of photon j. In particular, let us choose $A = X_i$, $a = Y_i$, $B = X_j$, and $b = Y_j$, where X_q and Y_q are the polarizations of photon q along the basis X and Y, respectively. The particular CHSH-Bell inequality (1) we are interested in is the one in which $m = n = y_k$, where y_k is one of the possible results, -1 or 1 (although we do

not know which one), of measuring Y_k. With this choice we obtain the CHSH-Bell inequality

$$|C(X_i, X_j) - y_k C(X_i, Y_j)$$
$$- y_k C(Y_i, X_j) - C(Y_i, Y_j)| \leqslant 2, \qquad (7)$$

which holds for local realistic theories, regardless of the particular value, either -1 or 1, of y_k.

We could force photons i and j to be those in locations 1 and 2, by measuring X on the photon in location 3, and then selecting only those events in which the result of this measurement is 1. This procedure guarantees that our definition of photons i and j is physically meaningful. However, it does not allow us to measure Y on photon k. The key point for testing inequality (7) is noticing that we do not need to know in which locations are photons i, j, and k for every three-photon system. We can obtain the required data by performing suitable combinations of measurements of X or Y on the three photons. In order to see this, let us first translate inequality (7) into the language of joint probabilities. Assuming that the expected value of any local observable cannot be affected by anything done to a distant particle, the CHSH-Bell inequality (7) can be transformed into a more convenient experimental inequality [11, 12]:

$$-1 \leqslant P_{X_i X_j}(-1, -1) - P_{X_i Y_j}(-1, -y_k)$$
$$- P_{Y_i X_j}(-y_k, -1) - P_{Y_i Y_j}(y_k, y_k) \leqslant 0. \qquad (8)$$

The bounds l of inequalities (1) and (7) are transformed into the bounds $(l - 2)/4$ of inequality (8). Therefore, the local realistic bound in (8) is 0, Cirel'son's bound is $(\sqrt{2} - 1)/2$, and the maximum value is $1/2$.

To measure the inequality (8), we must relate the four joint probabilities appearing in (8) to the probabilities of coincidences in a experiment with three spatial locations, 1, 2, and 3. For instance, it can be easily seen that

$$P_{X_i X_j}(-1, -1) =$$
$$P_{X_1 X_2 X_3}(1, -1, -1) + P_{X_1 X_2 X_3}(-1, 1, -1)$$
$$+ P_{X_1 X_2 X_3}(-1, -1, 1) + P_{X_1 X_2 X_3}(-1, -1, -1). \quad (9)$$

In addition, $P_{X_i Y_j}(-1, -y_k)$ and $P_{Y_i X_j}(-y_k, -1)$ are both less than or equal to

$$P_{X_1 Y_2 Y_3}(-1, 1, -1) + P_{X_1 Y_2 Y_3}(-1, -1, 1)$$
$$+ P_{Y_1 X_2 Y_3}(1, -1, -1) + P_{Y_1 X_2 Y_3}(-1, -1, 1)$$
$$+ P_{Y_1 Y_2 X_3}(1, -1, -1) + P_{Y_1 Y_2 X_3}(-1, 1, -1). \quad (10)$$

Finally,

$$P_{Y_i Y_j}(y_k, y_k) = P_{Y_1 Y_2 Y_3}(1, 1, 1) + P_{Y_1 Y_2 Y_3}(-1, -1, -1). \qquad (11)$$

Therefore, by performing measurements in 5 specific configurations (XXX, XYY, XYX, YXX, and YYY), we can obtain the value of the middle side of inequality (8).
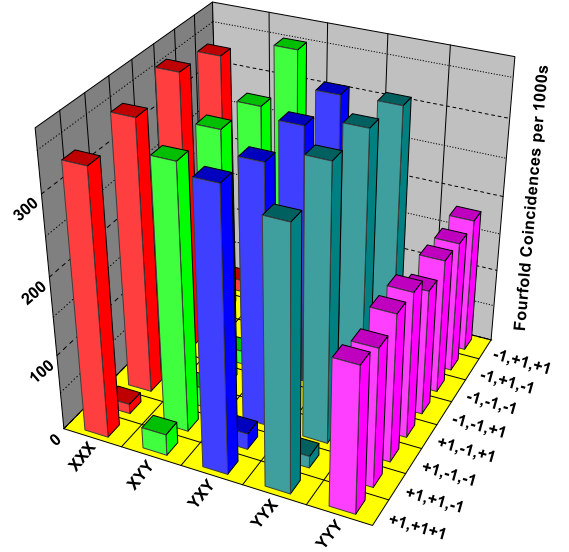

FIG. 3: Experimental results observed for the 5 required configurations: XXX, XYY YXY, YYX, and YYY.

In the state (3), the first three probabilities in the right-hand of (9) are expected to be $1/4$, and the fourth is expected to be zero; the six probabilities in (10) are expected to be zero, and the two probabilities in the right hand side of (11) are expected to be $1/8$. Therefore, the middle side of inequality (8) is expected to be $1/2$, which means that the left-hand side of inequality (7) is 4, which is not only beyond Cirel'son's bound, $2\sqrt{2}$, but is also the maximum possible violation of inequality (7).

The experiments consists of performing measurements in 5 specific configurations. As shown in Fig. 1, we use polarizers oriented at $\pm 45^0$ and $\lambda/4$ plates to perform X or Y measurements. For these 5 configurations, the experimental results for all possible outcomes are shown in Fig. 3.

Substituting the experimental results (shown in Fig. 3) into the right-hand side of (9), we obtain

$$P_{X_i X_j}(-1, -1) = 0.738 \pm 0.012. \qquad (12)$$

Similarly, substituting the experimental results in (10), we obtain

$$P_{X_i Y_j}(-1, -y_k) \leqslant 0.072 \pm 0.007,$$
$$P_{Y_i X_j}(-y_k, -1) \leqslant 0.072 \pm 0.007. \qquad (13)$$

Finally, substituting the experimental results in (11), we obtain

$$P_{Y_i Y_j}(y_k, y_k) = 0.254 \pm 0.011. \qquad (14)$$

Therefore, the prediction for the middle side of (8) is greater than or equal to 0.340 ± 0.019, and the prediction for the right-hand side of (7) is greater than or equal to

3.36 ± 0.08, which clearly violates Cirel'son's bound by 7 standard deviations.

In addition, using part of the results contained in Fig. 3, we can test the three-particle Bell inequality derived by Mermin [8],

$$|C(X_1, Y_2, Y_3) + C(Y_1, X_2, Y_3)$$
$$+ C(Y_1, Y_2, X_3) - C(X_1, X_2, X_3)| \leqslant 2. \qquad (15)$$

From the results in Fig. 3 we obtain 3.57 ± 0.04 for the left-hand side of (15), which is a violation of inequality (15) by 39 standard deviations. Note that the experiment for observing the violation beyond Cirel'son's bound also requires performing measurements in an additional configuration (YYY).

In conclusion, we have demonstrated a violation of the CHSH-Bell inequality beyond Cirel'son's bound. It should be emphasized that such a violation is predicted by quantum mechanics and appears when examining the data from the perspective of local realistic theories [6]. In addition, it should be stressed that the reported experiment is different as previous experiments to test local realism involving three or four-qubit GHZ states [13, 14], since it is based on a definition of pairs which is conditioned to the result of a measurement on a third qubit, and requires performing measurements in additional configurations.

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[1] A. Peres, *Quantum Theory: Concepts and Methods* (Kluwer, Dordrecht, 1993).
[2] J.S. Bell, Physics (Long Island City, NY) **1**, 195 (1964).
[3] J.S. Bell, *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, Cambridge, 1987).
[4] J.F. Clauser, M.A. Horne, A. Shimony, and R.A. Holt, Phys. Rev. Lett. **23**, 880 (1969).
[5] B.S. Cirel'son, Lett. Math. Phys. **4**, 93 (1980).
[6] A. Cabello, Phys. Rev. Lett. **88**, 060403 (2002).
[7] D.M. Greenberger, M.A. Horne, and A. Zeilinger, in *Bell's Theorem, Quantum Theory, and Conceptions of the Universe*, edited by M. Kafatos (Kluwer, Dordrecht, 1989), p. 69.
[8] N.D. Mermin, Phys. Rev. Lett. **65**, 1838 (1990).
[9] D. Bouwmeester, J.-W. Pan, M. Daniell, H. Weinfurter, and A. Zeilinger, Phys. Rev. Lett. **82**, 1345 (1999).
[10] J.-W. Pan, M. Daniell, S. Gasparoni, G. Weihs, and A. Zeilinger, Phys. Rev. Lett. **86**, 4435 (2001).
[11] J.F. Clauser and M. A. Horne, Phys. Rev. D **10**, 526 (1974).
[12] N.D. Mermin, Ann. N.Y. Acad. Sci. **755**, 616 (1995).
[13] J.-W. Pan, D. Bouwmeester, M. Daniell, H. Weinfurter, and A. Zeilinger, Nature **403**, 515 (2000).
[14] Z. Zhao, T. Yang, Y.-A. Chen, A.-N. Zhang, M. Żukowski, and J.-W. Pan, Phys. Rev. Lett. **91**, 180401 (2003).